

AmirHosein Sadeghimanesh
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This worksheet find the distance of the parameter point (0.08, 0.02) from the boundary of the multistationarity region in Section 4.4 of the paper.

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> restart:
n := 2: # Dimension of the parameter region.
B := Matrix([[0, 0.1], [0, 0.1]]): # The parameter region.
a := [1.000973, 1.027382, 0.975112, -110.368722, 103.486050,
-33.555818]: # Coefficients of the PSS polynomial.
monomialsVec := [1, x[1], x[2], x[1]^2, x[1]*x[2], x[2]^2]:
p := add(a[i]*monomialsVec[i], i = 1..6): # This is the PSS
polynomial.
kStar := [0.08, 0.02]: # This is the parameter point to
approximate its distance with the multistationarity region's
boundary.
F := add((x[i]-kStar[i])^2, i = 1..2) + lambda*(p-1): # The
auxiliary function of the Lagrangian multipliers.
eqns := [seq(diff(F, x[i]), i = 1..2), diff(F, lambda)]:
st := time[real]():
Sol := solve(eqns):
printf("The command \"solve\" from Maple took %a seconds to find
all complex solutions to the given system of equations.\n", time
[real]()-st);
printf("There are %a (possibly complex) solutions.\n", numelems(
[Sol]));
kPoints := Array([]):
ArrayTools:-Extend(kPoints, [seq([0, 0], i = 1..numelems([Sol]))]
):
for i from 1 by 1 to numelems([Sol]) do
  kPoints[i] := eval([x[1], x[2]], [Sol][i]): # The i-th solution.
end do:
kPoints := remove[flatten](j -> j[1] < B[1, 1] or j[1] > B[1, 2]
or j[2] < B[2, 1] or j[2] > B[2, 2], kPoints): # removing the
points that are not inside B.
printf(cat("Only the following points are inside the region of
interest.\n", seq(cat(j, "\n"), j in kPoints)));
distList := [seq(sqrt(add((j[i]-kStar[i])^2, i = 1..n)), j in
kPoints)]: # A list containing the distances of kStar from the
points in kPoints.
minDist := min(distList):
minIdx := min[index](distList):
printf("The minimum is obtained at k = %a which is equal to %a.",
kPoints[minIdx], minDist);
```

The command solve from Maple took .167 seconds to find all
complex solutions to the given system of equations.
There are 4 (possibly complex) solutions.
Only the following points are inside the region of interest.
[.4653097826e-2, .4560758310e-1]
[.4499222669e-1, .4161251428e-1]
The minimum is obtained at k = [.4499222669e-1, .4161251428e-1]
which is equal to .4114176669e-1.

| End of the file.
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